On the Stationary Solution of the Mathematical Model for Grain Boundary Grooving (Functional Equations in Mathematical Models)

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1. Introduction

In this talk, we will present some stationary solution for nonlinear partial differential equation called Mullins Equation which is occered in the theory of grain boundary grooving.

$$u_t = -C_1^E(u)(1+u_x^2)^{1/2} exp(-C_2^E(u)\frac{u_{xx}}{(1+u_x^2)^{3/2}}) + C_1^C(u)(1+u_x^2)^{1/2}. \quad (1)$$

The main tool, which we can use, is the admissibility property between weighted continuous function spaces for the integral operator,as follows.

$$T_\xi x(t) = -\int_t^\infty e^{\zeta_1(t-s)} F(x(s), y(s)) ds,$$

$$T_\xi y(t) = \xi e^{\zeta_2 t} + \int_0^t e^{\zeta_2(t-s)} F(x(s), y(s)) ds. \quad (2)$$

From this admissibility we can prove the existence theorem for the special simultaneous differential equation. This existence theorem can be applied for the second order differential equation,

$$u'' = f(u, u') = \frac{kT(u)(1+u'^2)^{3/2}}{v\gamma} ln(\frac{P_0(u)}{P_c}). \quad (3)$$

The solution of this equation is one of the stationary solution for Mullins Equa-

2. Theorems

On the equation (1), we are interested in the stational solution. So we shall consider the equation (3) which we can make by putting $u_t = 0$ for the equation (1). To prove the existence theorem for the stational solution, we use the next two theorems.

Theorem1

For the second oreder differential equation,

$$u'' = f(u, u'), \quad (4)$$

suppose that the following hypotheses.

$$f(u,p) \in C^1(R^2), \quad x > 0, \quad {}^{\exists}\lambda \in R^1 \quad s.t \quad .f(\lambda, 0) = 0, \quad f_u(\lambda, 0) > 0$$

.

Then there exits the solution on $(0, \infty)$ and it satisfies that

$${}^{\exists}D > 0 \quad s.t. \quad |u(x) - \lambda| \leq Dexp(-\tau x),$$

where

$$0 < \tau < \left| \frac{f_p(\lambda, 0) - \sqrt{f_p(\lambda, 0)^2 + 4f_u(\lambda, 0)}}{2} \right|.$$

Theorem2

On the differential equation,

$$\omega_1' = \zeta_1\omega_1 + F(\omega_1, \omega_2), \quad \omega_2' = \zeta_2\omega_2 + F(\omega_1, \omega_2), \quad x > 0,$$

where,

$$f(\eta_1, \eta_2) \in C^1(R^2), \quad F(0,0) = 0, \quad F_{\eta_1}(0,0) = 0, \quad \zeta_1 > 0, \zeta_2 < 0,$$

there exists some global nontrivial solution

$$\omega(x) = (\omega_1(x), \omega_2(x)), \quad x > 0,$$

for every $\tau, 0 < \tau < |\zeta_2|$, and the next inequality is satisfied.

$$|e^{\tau x}\omega_1(x)| + |e^{\tau x}\omega_2(x)| < \infty, \quad x > 0.$$

At first we consider Theorem2. By using the addmissibility of the integral operator(2), we can establish the proof of Theorem2. Let consider the integral operator on the following function set B,

$$B = \omega(x) = (\omega_1(x), \omega_2(x)) \in C^0([0, \infty)); ||\omega|| \leq 2|\xi|,$$

$$||\omega|| = \sup_{x \geq 0}(e^{\tau x}\omega_1(x) + e^{\tau x}\omega_2(x)).$$

On this set the integral operator(2) satisfies the contraction princeple. Then the operator $T_\xi : B \longrightarrow B$ has the unique fixes point $\omega(x) = (\omega_1(x), \omega_2(x))$. Hence we can prove Theorem2. Next we treat Theoreml, by using the results of Theorem2. Let define the function $F(\omega_1, \omega_2)$ in Theorem2 by the next equation,

$$F(\eta_1, \eta_2) = f(\frac{\eta_1 - \eta_2}{\zeta_1 - \zeta_2} + \lambda, \frac{\zeta_1\eta_1 - \zeta_2\eta_2}{\zeta_1 - \zeta_2}) - \frac{\eta_1 - \eta_2}{\zeta_1 - \zeta_2} f_u(\lambda, 0) - \frac{\zeta_1\eta_1 - \zeta_2\eta_2}{\zeta_1 - \zeta_2} f_p(\lambda, 0),$$

where

$$\zeta_1 = \frac{f_p(\lambda, 0) + \sqrt{f_p(\lambda, 0)^2 + 4f_u(\lambda, 0)}}{2} > 0,$$

$$\zeta_2 = \frac{f_p(\lambda, 0) - \sqrt{f_p(\lambda, 0)^2 + 4f_u(\lambda, 0)}}{2} < 0,$$

where the function f as in Theoreml. By the result of Theorem2 there exists the solution $\omega(x) = (\omega_1(x), \omega_2(x))$. Define

$$u(x) = \frac{\omega_1(x) - \omega_2(x)}{\zeta_1 - \zeta_2} + \lambda, \quad x > 0.$$

This function u is the solution in Theoreml. At last, we can apply Theoreml for the equation (3), we get the stational solution of (1).

References

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